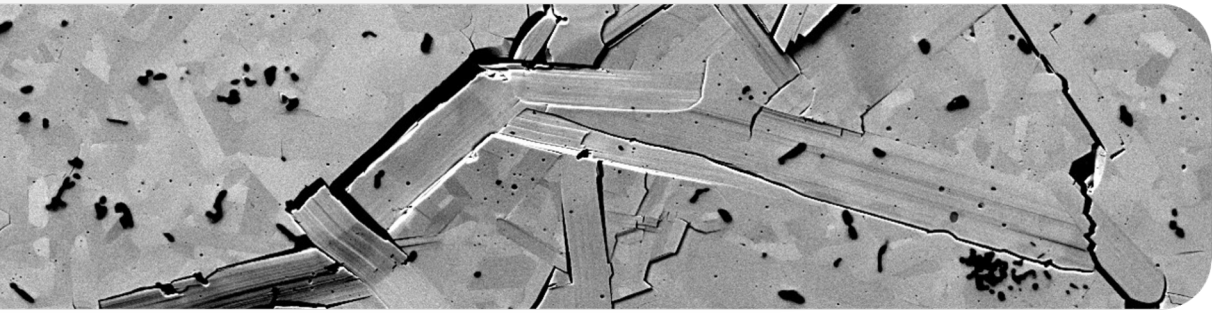
COE–3001: Mechanics of deformable bodies

Chapter 7: Principle of superposition and static indeterminacy

Prof. Antoine GUITTON

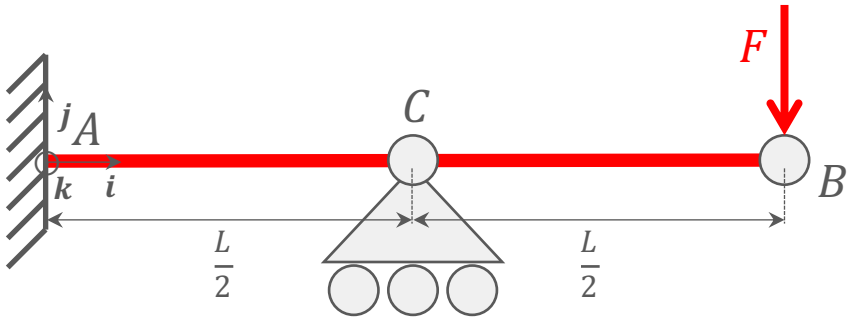
Université de Lorraine, CNRS, Arts et Métiers Institute of Technology, LEM3, F-57000 Metz,
antoine.guitton@univ-Lorraine.fr

© Antoine Guitton, Université de Lorraine, 2026. All rights reserved. For educational use only.
Any reproduction, distribution, or commercial use without prior written permission is prohibited.



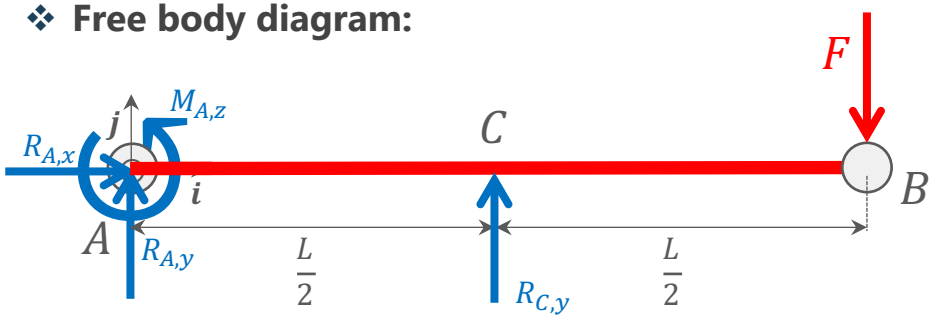
Hyperstatism

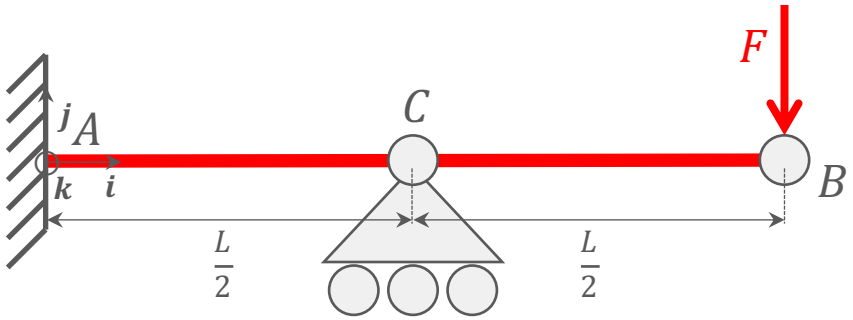
Presentation of the problem



Presentation of the problem

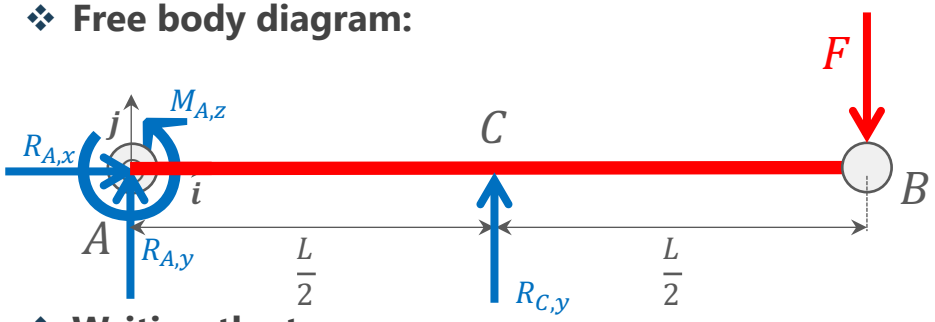
❖ **Free body diagram:**





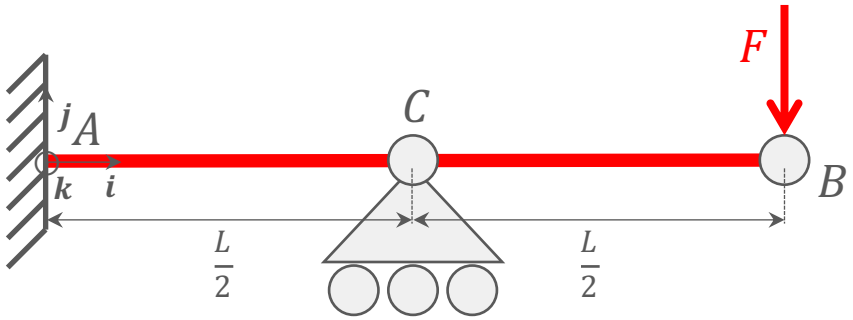
Presentation of the problem

❖ **Free body diagram:**



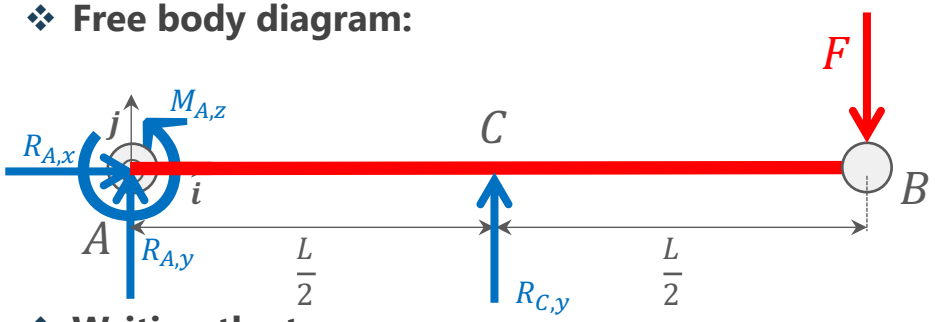
❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}; \{\mathcal{T}_C\} = \begin{Bmatrix} 0 & 0 \\ R_{C,y} & 0 \\ 0 & 0 \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & 0 \end{Bmatrix}$$



Presentation of the problem

❖ **Free body diagram:**



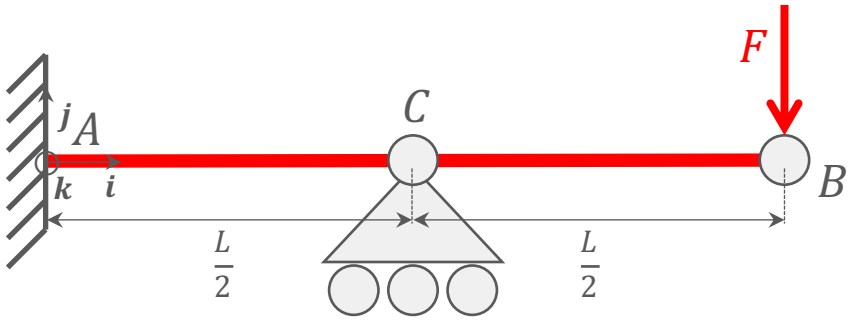
❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}; \{\mathcal{T}_C\} = \begin{Bmatrix} 0 & 0 \\ R_{C,y} & 0 \\ 0 & 0 \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & 0 \end{Bmatrix}$$

❖ **Let's apply the FPS in A:**

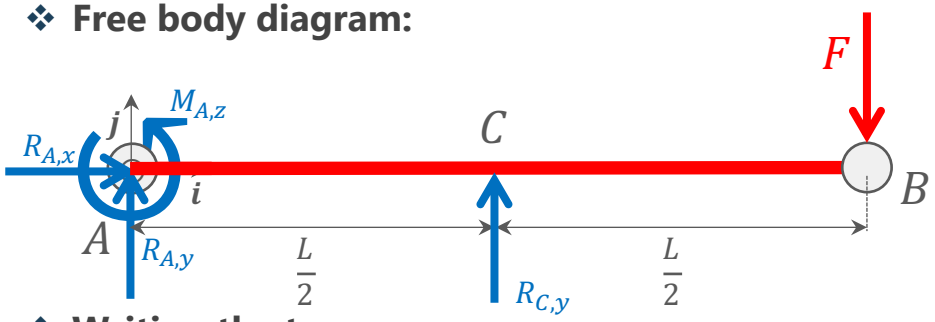
$$FPS \Rightarrow \begin{cases} R_{A,x} = 0 \\ R_{A,y} + R_{C,y} - F = 0 \\ M_{A,z} + \dfrac{LR_{C,y}}{2} - FL = 0 \end{cases}$$

↳ System of 2 equations with 3 unknowns
⇒ **Impossible to solve!**



Presentation of the problem

❖ **Free body diagram:**



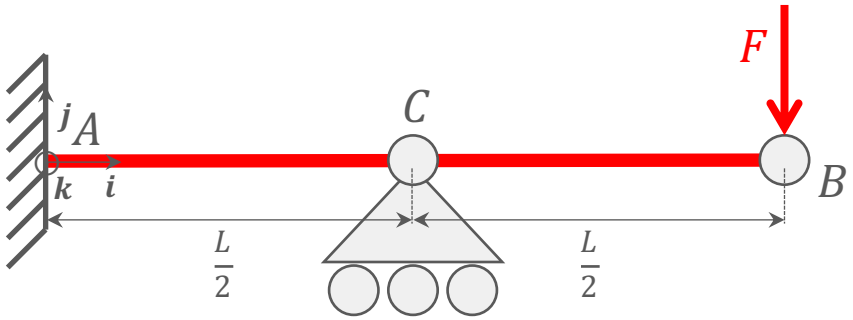
❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}; \{\mathcal{T}_C\} = \begin{Bmatrix} 0 & 0 \\ R_{C,y} & 0 \\ 0 & 0 \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & 0 \end{Bmatrix}$$

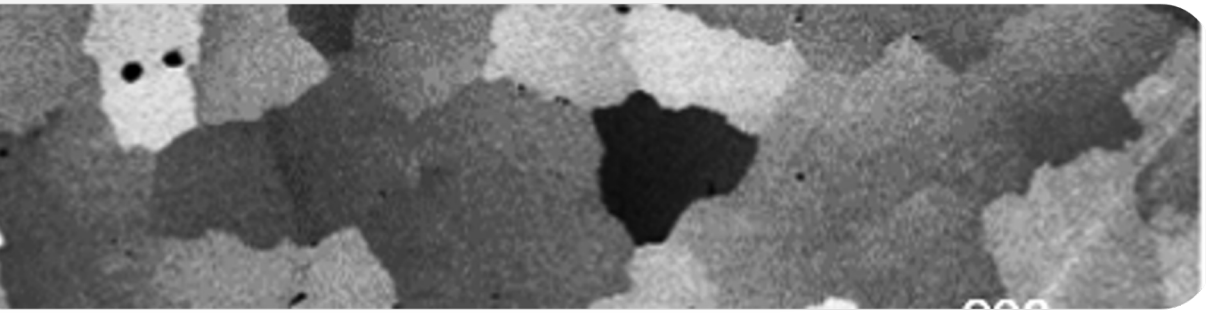
❖ **Let's apply the FPS in A:**

$$FPS \Rightarrow \begin{cases} R_{A,x} = 0 \\ R_{A,y} + R_{C,y} - F = 0 \\ M_{A,z} + \dfrac{LR_{C,y}}{2} - FL = 0 \end{cases}$$

↳ System of 2 equations with 3 unknowns
⇒ **Impossible to solve!**



↳ **Additional equations must be found.**



Solution using the principle of superposition

www.antoine-guitton.fr

Principle of superposition

- Applies to linear systems (linear material behavior and small deformations)
- The total response of a structure is the sum of the responses to each load taken separately
- Each load case is analyzed independently
- Boundary conditions must remain unchanged for all load cases

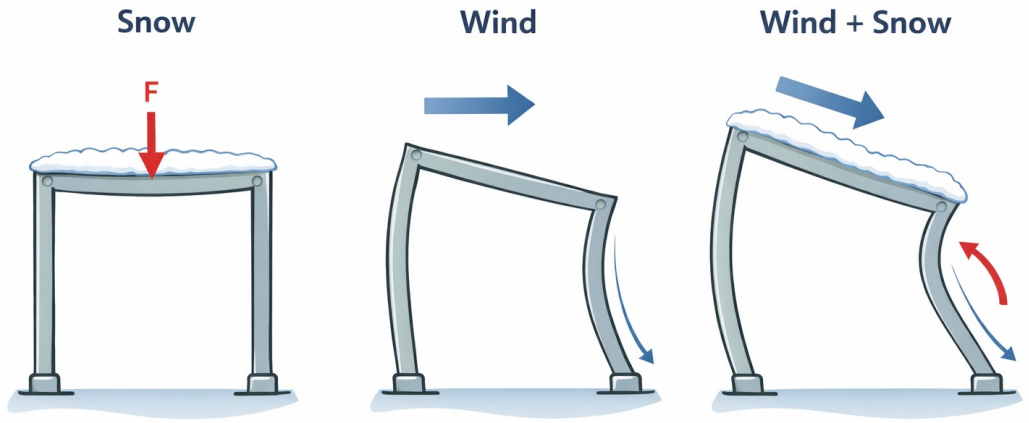
Principle of superposition

- **Applies to linear systems (linear material behavior and small deformations)**
- **The total response of a structure is the sum of the responses to each load taken separately**
- **Each load case is analyzed independently**
- **Boundary conditions must remain unchanged for all load cases**
- **Valid for:**
  - Forces and moments
  - Displacements and rotations
  - Stresses and internal forces
- **Not valid if:**
  - Material behavior is nonlinear (plasticity, large deformations)
  - Boundary conditions depend on the load

Principle of superposition

- **Applies to linear systems (linear material behavior and small deformations)**
- **The total response of a structure is the sum of the responses to each load taken separately**
- **Each load case is analyzed independently**
- **Boundary conditions must remain unchanged for all load cases**
- **Valid for:**
  - Forces and moments
  - Displacements and rotations
  - Stresses and internal forces
- **Not valid if:**
  - Material behavior is nonlinear (plasticity, large deformations)
  - Boundary conditions depend on the load

- **Pushing a door**
  - One person pushes lightly, the door moves a little
  - Two people push at the same time → the door moves more
  - The total effect is the sum of both pushes
- **Forces on a structure**
  - A beam subjected to a force and a moment
  - The final deformation equals the sum of the deformation due to each load



Principle of superposition

The total response of a structure is the sum of the responses to each load taken separately

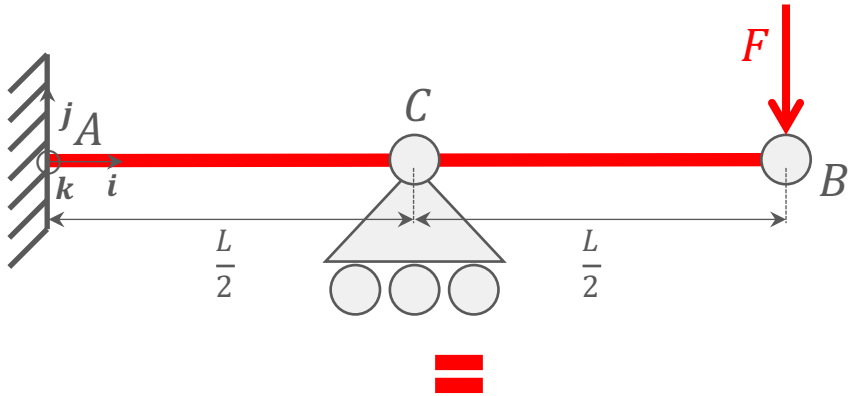
S_T

$=$

S_1

$+$

S_2



Principle of superposition

The total response of a structure is the sum of the responses to each load taken separately

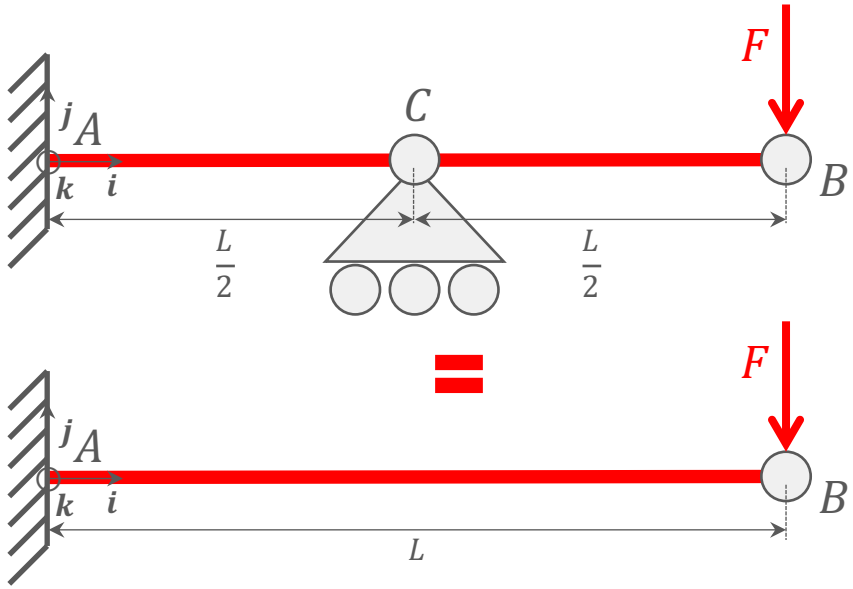
S_T

$=$

S_1

$+$

S_2



Principle of superposition

The total response of a structure is the sum of the responses to each load taken separately

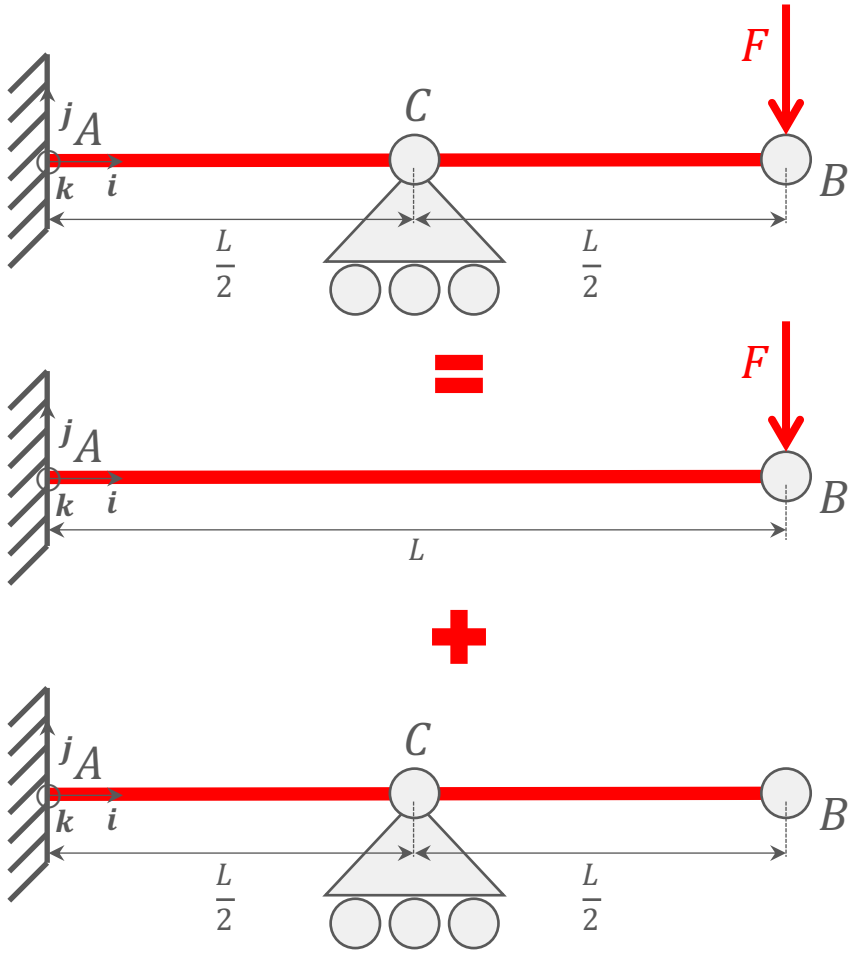
S_T

$=$

S_1

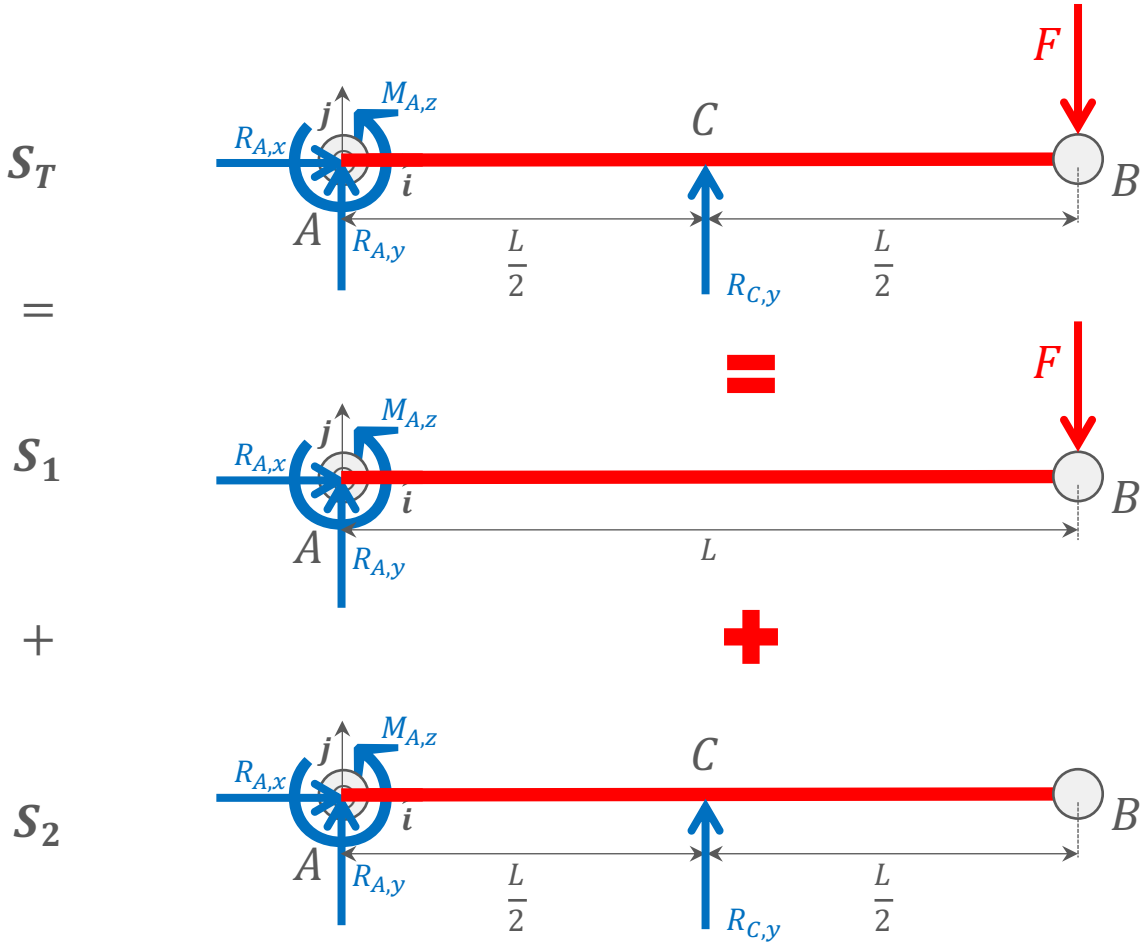
$+$

S_2

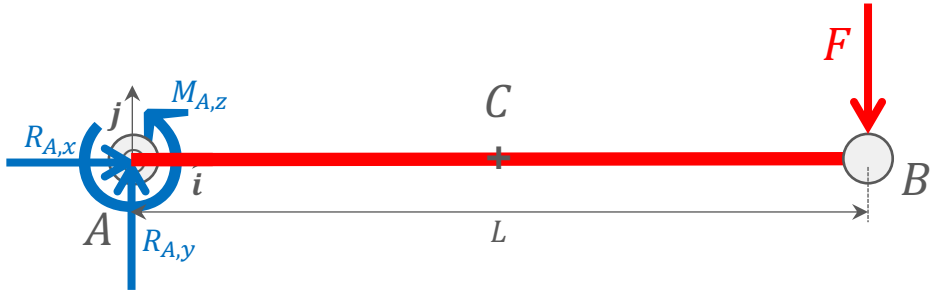


Principle of superposition

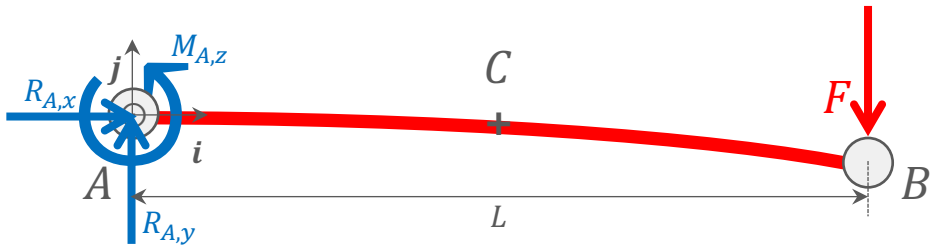
The total response of a structure is the sum of the responses to each load taken separately



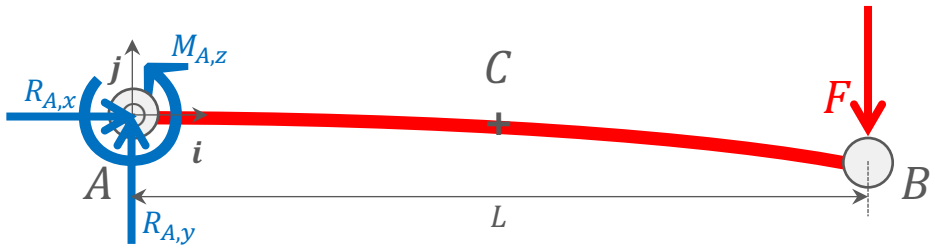
Solving S_1



Solving S_1



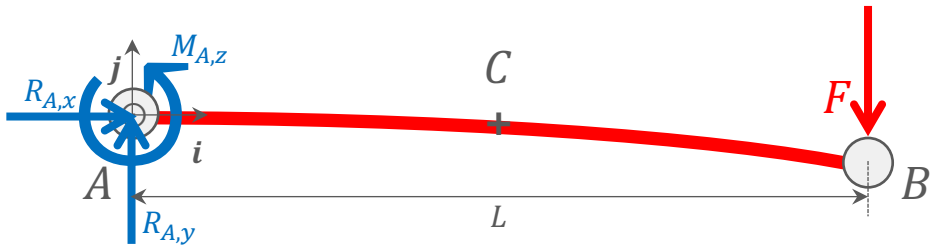
Solving S_1



❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & 0 \end{Bmatrix}$$

Solving S_1



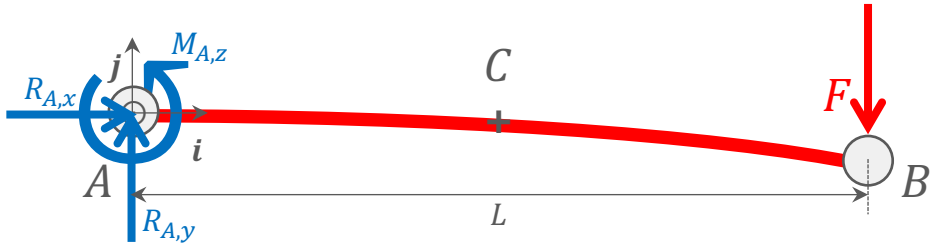
❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & 0 \end{Bmatrix}$$

❖ **Cohesion torsors:**

$$\{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & -LF \end{Bmatrix}; \{\mathcal{T}_X^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & -LF + xF \end{Bmatrix}$$

Solving S_1



❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}; \{\mathcal{T}_B\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & 0 \end{Bmatrix}$$

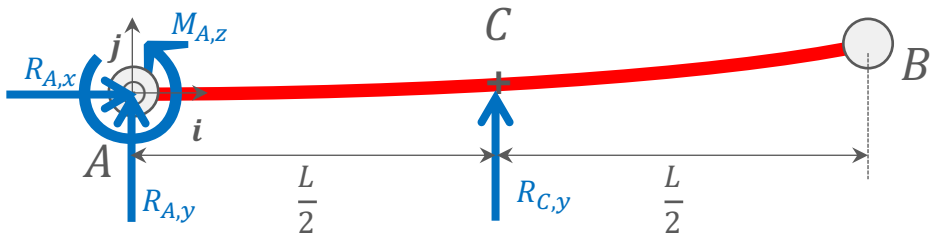
❖ **Cohesion torsors:**

$$\{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & -LF \end{Bmatrix}; \{\mathcal{T}_X^{coh}\} = \begin{Bmatrix} 0 & 0 \\ -F & 0 \\ 0 & -LF + xF \end{Bmatrix}$$

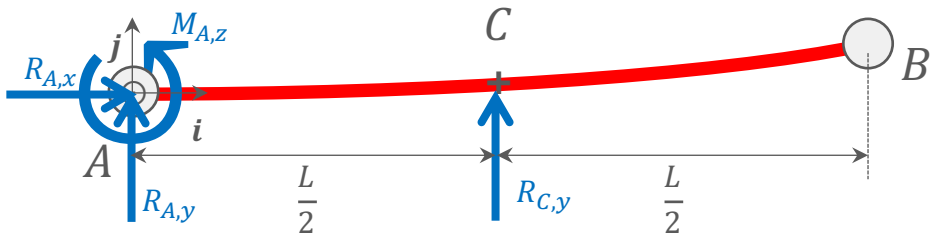
❖ **Deflection at $C(\frac{L}{2}, 0)$:**

$$v_{S_1}(C) = -\frac{5FL^3}{48EI_x}$$

Solving S_2



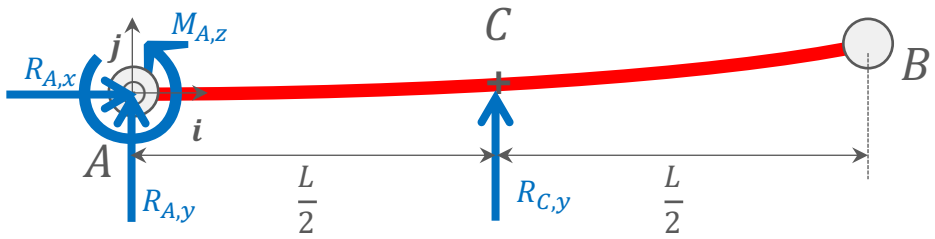
Solving S_2



❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ -R_{c,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}$$

Solving S_2



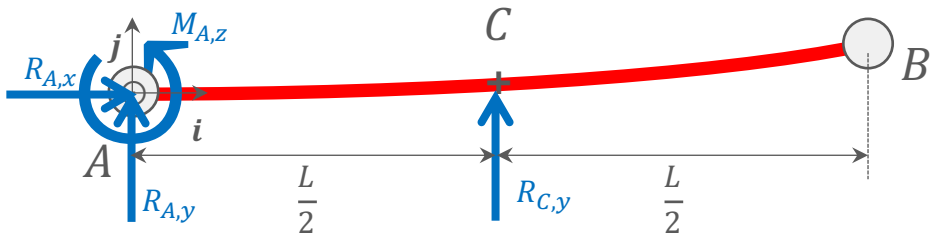
❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ -R_{c,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}$$

❖ **Cohesion torsors:**

$$\{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ R_{c,y} & 0 \\ 0 & -M_{A,z} \end{Bmatrix}; \{\mathcal{T}_X^{coh}\} = \begin{Bmatrix} 0 & 0 \\ R_{c,y} & 0 \\ 0 & -M_{A,z} - xR_{c,y} \end{Bmatrix}$$

Solving S_2



❖ **Writing the torsors:**

$$\{\mathcal{T}_A\} = \begin{Bmatrix} R_{A,x} & 0 \\ R_{A,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix} = \begin{Bmatrix} 0 & 0 \\ -R_{c,y} & 0 \\ 0 & M_{A,z} \end{Bmatrix}$$

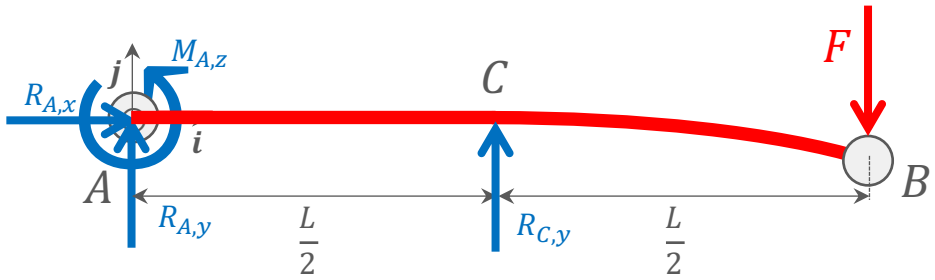
❖ **Cohesion torsors:**

$$\{\mathcal{T}_A^{coh}\} = \begin{Bmatrix} 0 & 0 \\ R_{c,y} & 0 \\ 0 & -M_{A,z} \end{Bmatrix}; \{\mathcal{T}_X^{coh}\} = \begin{Bmatrix} 0 & 0 \\ R_{c,y} & 0 \\ 0 & -M_{A,z} - xR_{c,y} \end{Bmatrix}$$

❖ **Deflection at $C(\frac{L}{2}, 0)$:**

$$v_{S_2}(C) = \frac{2R_{c,y}L^3}{48EI_x}$$

Solving S_T: applying the Principle of superposition

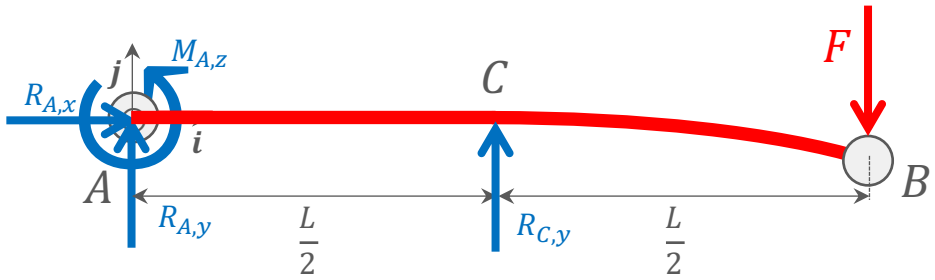


❖ **Deflection equations at $C(\frac{L}{2}, 0)$:**

$$v_{S_1}(C) + v_2(C) = 0$$

$$\Rightarrow -\frac{5FL^3}{48EI_x} + \frac{2R_{c,y}L^3}{48EI_x} = 0$$

Solving S_T: applying the Principle of superposition



❖ **Deflection equations at $C(\frac{L}{2}, 0)$:**

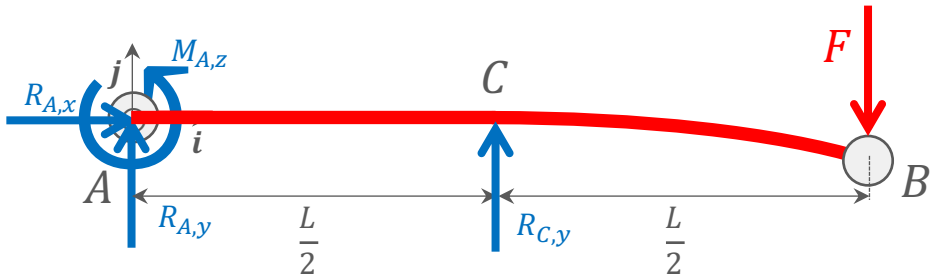
$$v_{S_1}(C) + v_2(C) = 0$$

$$\Rightarrow -\frac{5FL^3}{48EI_x} + \frac{2R_{c,y}L^3}{48EI_x} = 0$$

❖ **FPS:**

$$FPS \Rightarrow \begin{cases} R_{A,x} = 0 \\ R_{A,y} + R_{C,y} - F = 0 \\ M_{A,z} + \frac{LR_{C,y}}{2} - FL = 0 \end{cases}$$

Solving S_T: applying the Principle of superposition



❖ **Deflection equations at $C(\frac{L}{2}, 0)$:**

$$v_{S_1}(C) + v_2(C) = 0$$

$$\Rightarrow -\frac{5FL^3}{48EI_x} + \frac{2R_{c,y}L^3}{48EI_x} = 0$$

❖ **FPS:**

$$FPS \Rightarrow \begin{cases} R_{A,x} = 0 \\ R_{A,y} + R_{C,y} - F = 0 \\ M_{A,z} + \frac{LR_{C,y}}{2} - FL = 0 \end{cases}$$

↳ This is now a system of 3 equations with 3 unknowns
⇒ **Possible to solve!**

Thanks for your listening!

If you need further information:

Prof. Antoine GUITTON

Full Professor at Université de Lorraine

Phone (LEM3): +33 372 747 826

Email: antoine.guitton@univ-lorraine.fr

Website: www.antoine-guitton.fr

LABORATOIRE D'ÉTUDE DES MICROSTRUCTURES ET DE MÉCANIQUE DES MATÉRIAUX

UNIVERSITÉ DE LORRAINE

École Nationale d'Ingénieurs de Metz

Georgia Tech