Georgia Tech Europe

Mechanics of deformable bodies
COE – 3001
Bending
Homework #5

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Modeling assumptions

Unless otherwise stated, the following assumptions of strength of materials are adopted throughout this assignment:

- material is continuous, homogeneous, and isotropic,
- deformations are small,
- linear elastic behavior,
- stresses and strains are uniformly distributed,
- loads and supports are perfectly idealized.

Exercise I Questions on the course

Do you feel confident about your knowledge of shear loading? Please answer each question to assess your understanding.

1. In pure bending of a prismatic beam, the bending moment is constant along the beam segment.

☐ True ☐ False

2. The curvature of the neutral axis is defined as $\kappa = \dfrac{1}{R}$, where R is the radius of curvature.

☐ True ☐ False

3. Under the Navier–Bernoulli assumption, cross sections remain plane and perpendicular to the beam axis after bending.

☐ True ☐ False

4. In small deflection beam theory, the curvature can be approximated by $\kappa \approx \dfrac{d^2w}{dx^2}$.

☐ True ☐ False

5. The normal strain in bending varies linearly with the distance y from the neutral axis: $\varepsilon_x(y) = -\kappa y$.

☐ True ☐ False

6. The neutral axis is the line where the normal stress σ_x is maximum.

☐ True ☐ False

7. In elastic bending, the normal stress distribution across the section is linear: $\sigma_x(y) = -\frac{M}{I}y$.

☐ True ☐ False

8. The second moment of area I depends only on the beam material properties.

☐ True ☐ False

9. For a rectangular cross section of width b and height h (bending about the centroidal axis parallel to b), $I = \frac{bh^3}{12}$.

☐ True ☐ False

10. For a circular cross section of radius R, the second moment of area about a centroidal diameter is $I = \frac{\pi R^4}{4}$.

☐ True ☐ False

11. The maximum bending stress in a beam is $\sigma_{\max} = \frac{Mc}{I}$, where c is the distance to the farthest fiber.

☐ True ☐ False

12. If the bending moment M is doubled, the maximum normal stress $\sigma_{\max}$ is doubled (all else constant).

☐ True ☐ False

13. If the beam height h of a rectangular section is doubled, the second moment of area I is multiplied by 8.

☐ True ☐ False

14. In beam bending, shear stress is maximum at the outer surfaces of the beam.

☐ True ☐ False

15. In a rectangular beam, the shear stress distribution due to a transverse shear force V is parabolic, with a maximum at the neutral axis.

☐ True ☐ False

16. The transverse shear stress in a beam can be computed using $\tau = \frac{VQ}{Ib}$, where Q is the first moment of area.

☐ True ☐ False

17. The bending moment diagram is obtained by integrating the shear force diagram with respect to x.

☐ True ☐ False

18. The shear force diagram is obtained by differentiating the bending moment diagram: $V(x) = \dfrac{dM}{dx}$.

☐ True ☐ False

19. For a simply supported beam under a central point load P, the maximum deflection occurs at midspan.

☐ True ☐ False

20. In Euler–Bernoulli beam theory, increasing Young's modulus E decreases deflection for the same loading and geometry.

☐ True ☐ False

Exercise II Three-point bending test on an iPhone XR

A few years ago, shortly after the release of the iPhone XR, several users reported permanent bending of the device when it was placed in the back pocket of jeans and subjected to body weight while sitting.

This situation raised important engineering questions regarding bending stiffness, structural resistance, and elastic limits of thin multi-material consumer products.

In order to analyze this problem from a mechanics of materials perspective, a three-point bending test is performed on an iPhone XR of mass $m = 194\,\mathrm{g}$.

The phone is modeled as a beam resting on two supports separated by a span L and subjected to a central concentrated force F.

At each instant, the maximum deflection is recorded as a function of the applied force. The experimental force–deflection curve is shown below, where a linear fit has been performed on the initial (elastic) part of the curve.

Assume:

- The iPhone behaves as a homogeneous, isotropic beam in the elastic regime.
- Small deflections apply.
- Euler–Bernoulli beam theory is valid.

1. Draw the free body diagram (FBD) of the problem.

2. Determine the expressions of the internal force resultants (cohesion torsor) along the beam: normal force $N(x)$, shear force $V(x)$, and bending moment $M(x)$.

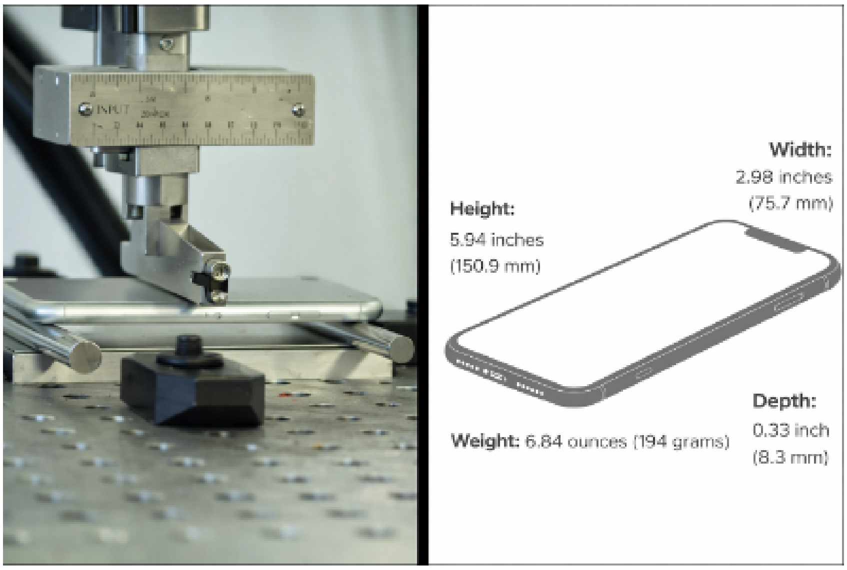

Figure 1: Three-point bending test performed on an iPhone XR. The device is simply supported at two points and loaded at midspan by a vertical concentrated force. The overall dimensions and mass of the phone are indicated.

3. Determine the equation of the deflection curve $w(x)$ as a function of the problem data.

4. Determine the expression of the maximum deflection δ as a function of the applied force F, the span L, Young's modulus E, and the second moment of area I.

5. Using the experimental linear fit in the elastic regime, determine the effective Young's modulus of the iPhone XR.

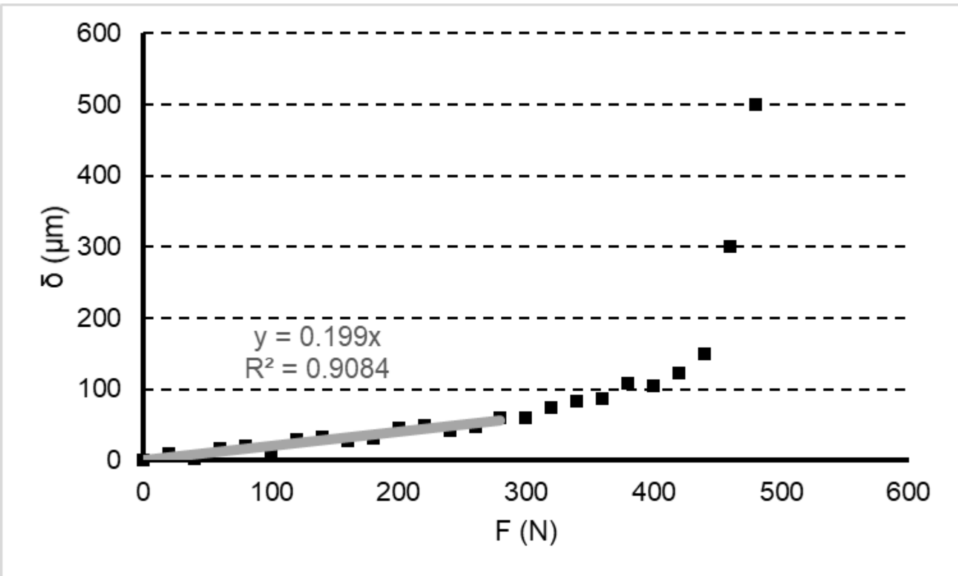

Figure 2: Experimental force–deflection curve obtained during the three-point bending test. The linear fit shown corresponds to the elastic regime and is used to determine the effective bending stiffness of the device.

Exercise III Wind Loading on a Cantilever Mailbox Post

During a storm, the mailbox (mass $m = 1\,\mathrm{kg}$) is subjected to wind pressure acting laterally on its side face, which can be modeled as a rectangular surface of dimensions $40\,\mathrm{cm} \times 30\,\mathrm{cm}$.

The maximum wind speed was measured at $V = 150\,\mathrm{km\,h^{-1}}$. The resulting wind force can be modeled by:

$$F = \frac{1}{2}\rho S V^2 C$$

with:

- ρ : air density, $\rho = 1.3\,\mathrm{kg\,m^{-3}}$,
- S : exposed area in $\mathrm{m^2}$,
- V : wind speed in $\mathrm{m\,s^{-1}}$,
- C : aerodynamic coefficient, here $C = 1.1$ (dimensionless).

The mailbox is fixed to the ground using a hollow circular tube (cantilever post) of outer diameter $D = 50\,\mathrm{mm}$, thickness $2e = 2\,\mathrm{mm}$ (so $e = 1\,\mathrm{mm}$), and length $L = 1.5\,\mathrm{m}$. The material properties are: $R_e = 200\,\mathrm{MPa}$ and $E = 100\,\mathrm{GPa}$.

1. Draw the free body diagram (FBD) of the system. Compute the magnitudes of the forces acting on the structure.
2. Determine the expressions of the internal force resultants (cohesion torsor) in the tube: normal force $N(x)$, shear force $V(x)$, and bending moment $M(x)$.
3. Compute the maximum stresses inside the post.
4. Compute the tip deflection of the mailbox.