Mechanics of deformable bodies
COE – 3001
Stress concentration
Homework #7

Prof Antoine GUITTON
antoine.guitton@univ-lorraine.fr

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Modeling assumptions

Unless otherwise stated, the following assumptions of strength of materials are adopted throughout this assignment:

- material is continuous, homogeneous, and isotropic,
- deformations are small,
- linear elastic behavior,
- stresses and strains are uniformly distributed,
- loads and supports are perfectly idealized.

Exercise I Power transmission through a stepped shaft

A stepped shaft rotates at $n = 900$ r/min. The allowable shear stress is $\tau_{\max} = 55\,\text{MPa}$. The shaft geometry is characterised by a shoulder fillet of radius r.

Assume:

- The shaft is subjected to pure torsion.
- The stress concentration factor K_t is read from the standard abacus for a stepped shaft in torsion (see Figure 1.
- The material behaves as homogeneous, isotropic, and linearly elastic.

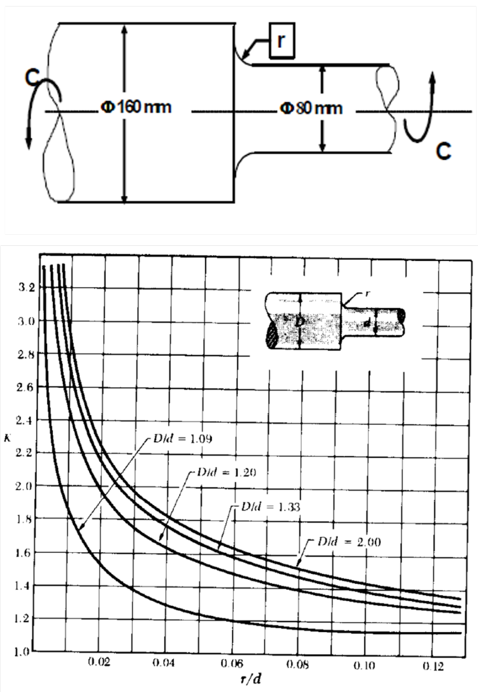

Figure 1: Stress concentration coefficient.

1. For a fillet radius $r = 4\,\text{mm}$, read the stress concentration factor K_t from the abacus. Deduce the allowable nominal shear stress τ_{nom}, then compute the maximum torque T the shaft can transmit. Conclude by calculating the transmitted power P in kW.

2. For a fillet radius $r = 10\,\text{mm}$, repeat the procedure. Compute the new transmitted power P' and determine the gain or loss in power with respect to question **1**. Comment on the effect of the fillet radius on the load-carrying capacity.

Exercise II Stress concentration around a circular hole – Kirsch solution

An infinite thin plate of thickness $t = 5\,\text{mm}$ contains a small circular hole of radius $a = 4\,\text{mm}$. The plate is subjected to a remote uniaxial tensile stress $\sigma_\infty = 80\,\text{MPa}$ applied along the x-axis. The material is linearly elastic with yield strength $\sigma_y = 250\,\text{MPa}$.

The Kirsch solution gives the hoop stress $\sigma_{\theta\theta}$ at the hole boundary ($r = a$):

$$\sigma_{\theta\theta}(a, \theta) = \sigma_\infty\,(1 - 2\cos 2\theta) \tag{1}$$

Assume:

- Plane stress conditions apply.
- The plate is infinite: edge effects are neglected.
- Small strains and linear elasticity hold throughout.

1. Determine the positions θ at which $\sigma_{\theta\theta}$ is maximum and minimum. Compute the corresponding stress values and deduce the theoretical stress concentration factor K_t.

2. At $\theta = 90°$, compute the maximum stress and check whether the material remains elastic. Define the load σ_∞^* at which first yielding occurs at the hole boundary.

3. The plate is now subjected to equibiaxial tension: $\sigma_x^\infty = \sigma_y^\infty = \sigma_\infty$. Using the Kirsch solution for this loading state:

$$\sigma_{\theta\theta}(a, \theta) = 2\sigma_\infty \tag{2}$$

compute K_t and compare with the uniaxial case. Interpret physically the absence of θ-dependence.

4. The remote stress is now pure shear: $\tau_{xy}^\infty = \tau_0 = 40\,\text{MPa}$, giving:

$$\sigma_{\theta\theta}(a, \theta) = -4\tau_0 \cos 2\theta \tag{3}$$

Determine the maximum hoop stress, the corresponding K_t (defined with respect to τ_0), and the angle θ at which it occurs.