COE–3001: Mechanics of deformable bodies

Chapter 7: Principle of superposition and static indeterminacy

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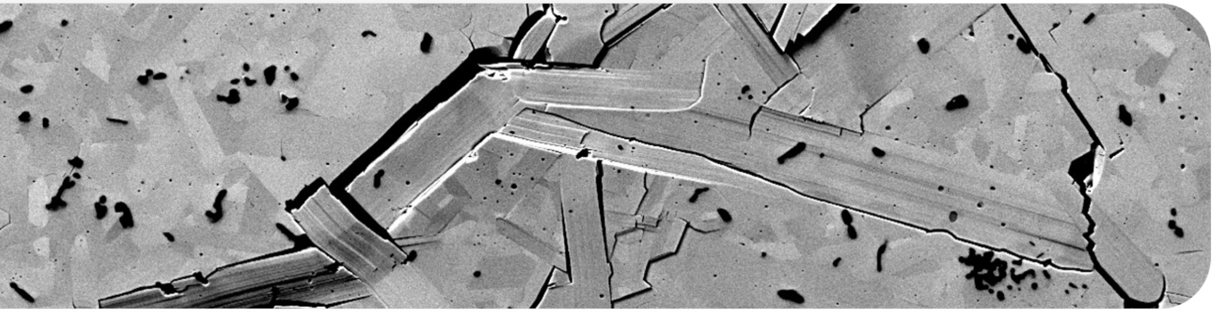
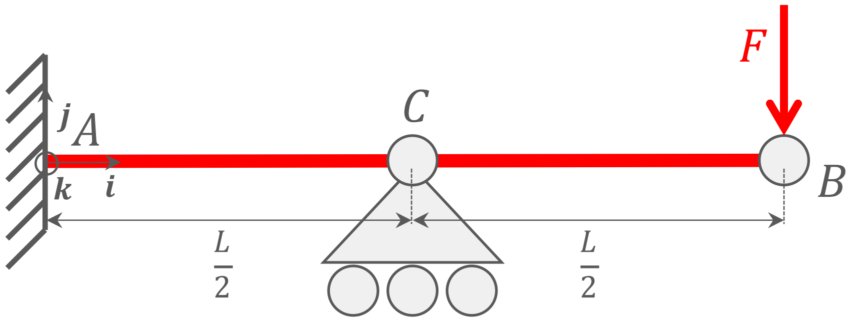


Hyperstatism

Presentation of the problem



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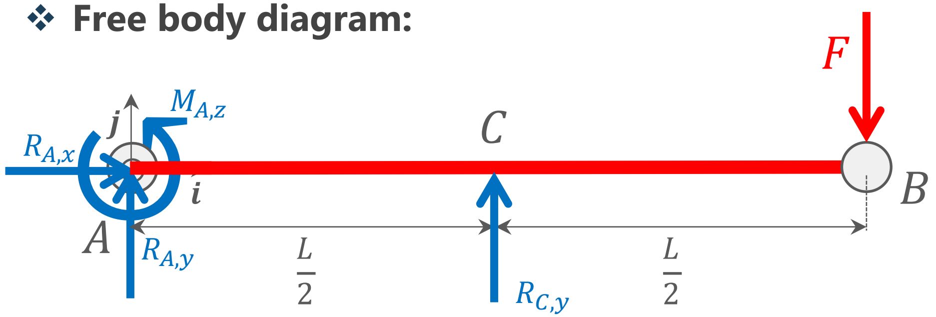

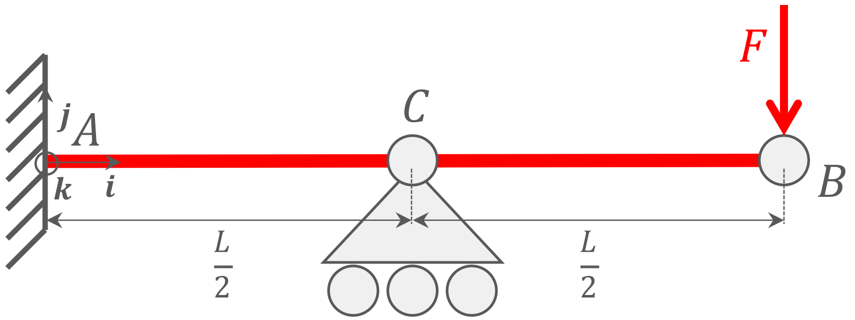

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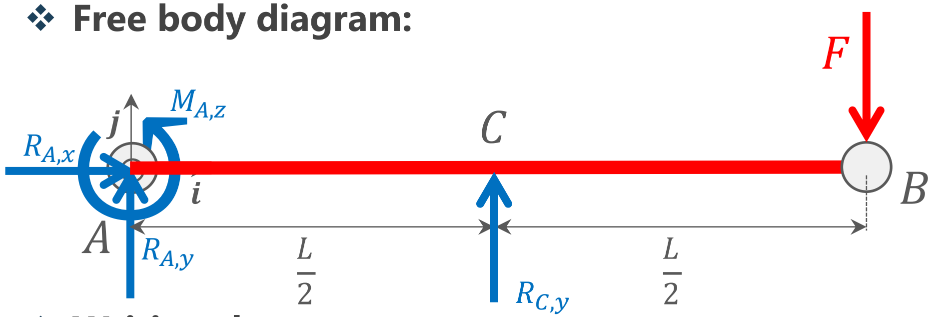

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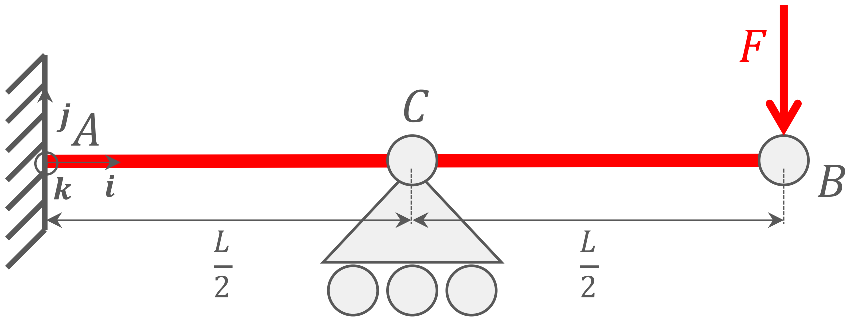

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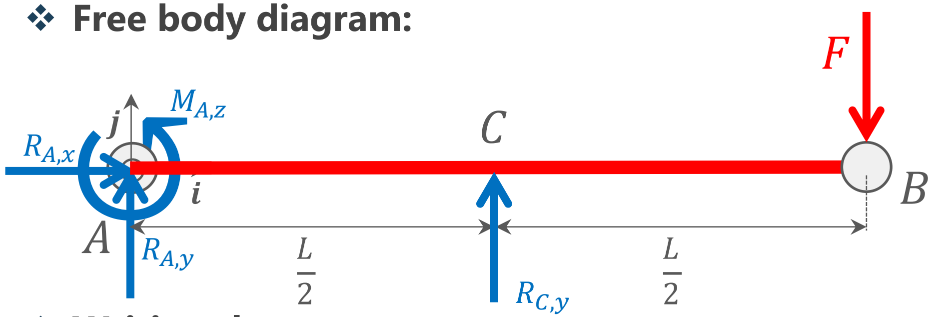

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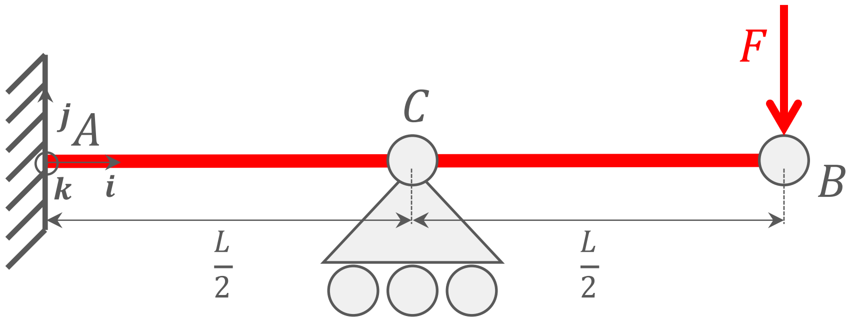

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⇒ **Impossible to solve!**

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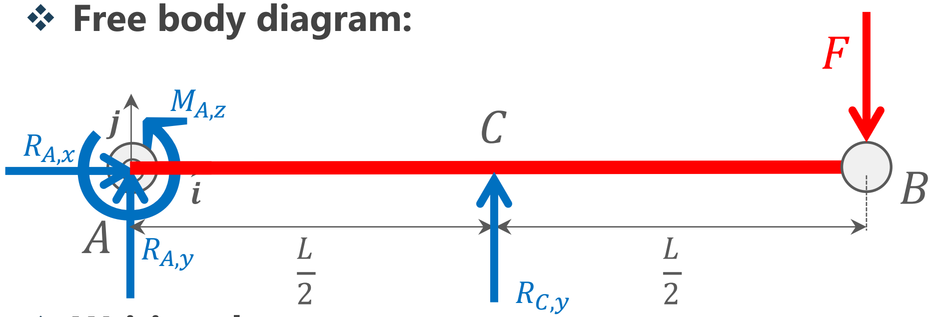

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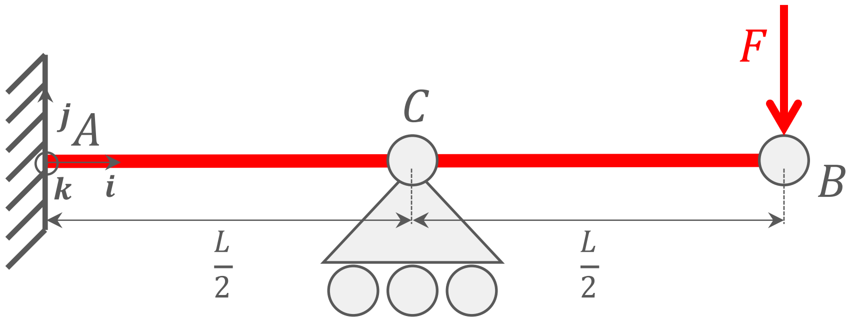
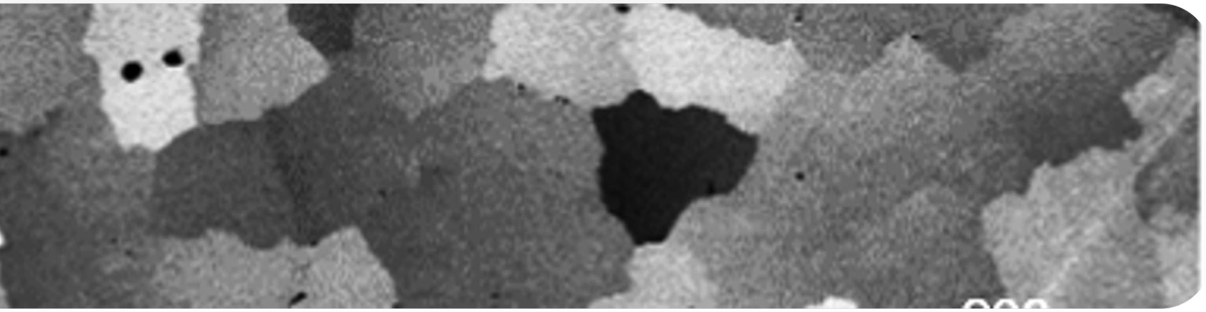


↳ **Additional equations must be found.**



Solution using the principle of superposition

Principle of superposition

- Applies to linear systems (linear material behavior and small deformations)
- The total response of a structure is the sum of the responses to each load taken separately
- Each load case is analyzed independently
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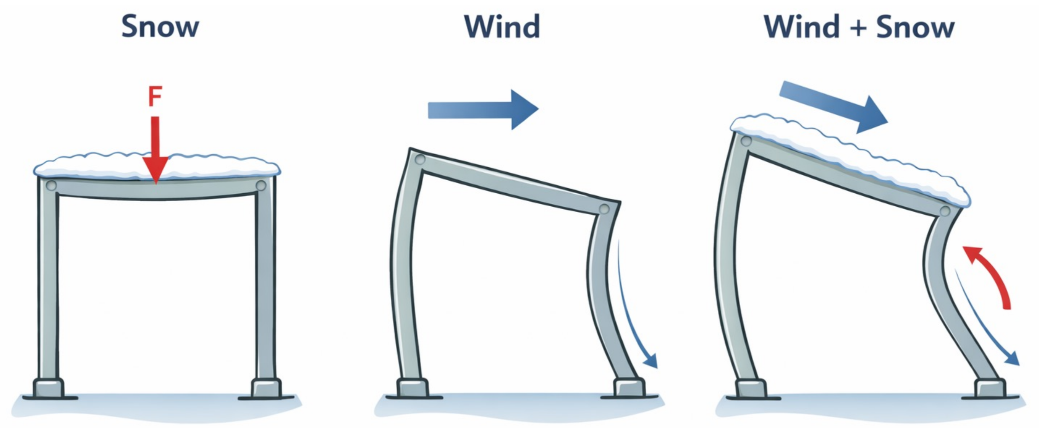
- **Pushing a door**
  - One person pushes lightly, the door moves a little
  - Two people push at the same time → the door moves more
  - The total effect is the sum of both pushes
- **Forces on a structure**
  - A beam subjected to a force and a moment
  - The final deformation equals the sum of the deformation due to each load



Principle of superposition

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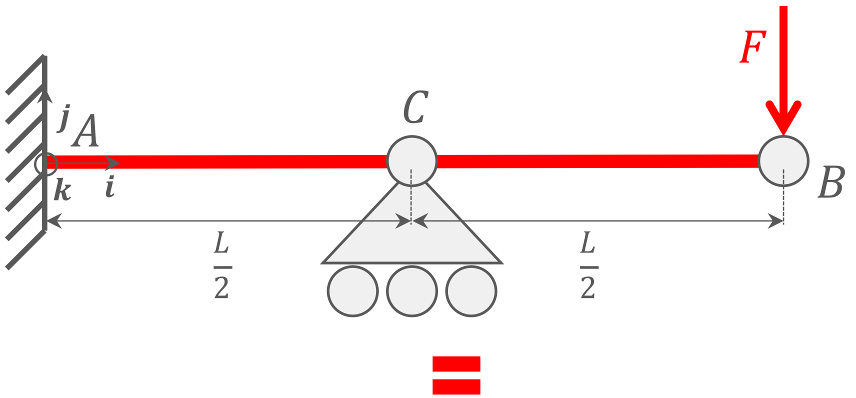
S_T



$=$

S_1

$+$

S_2

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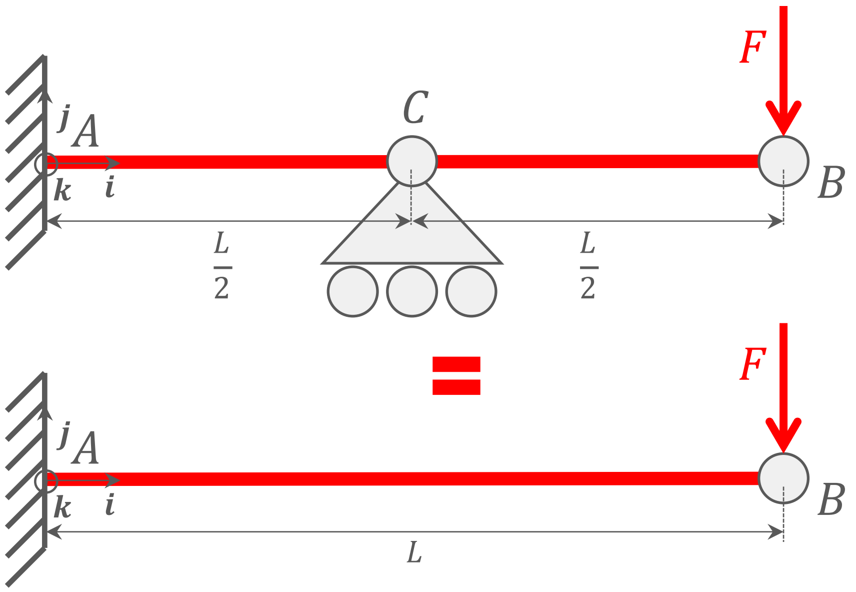

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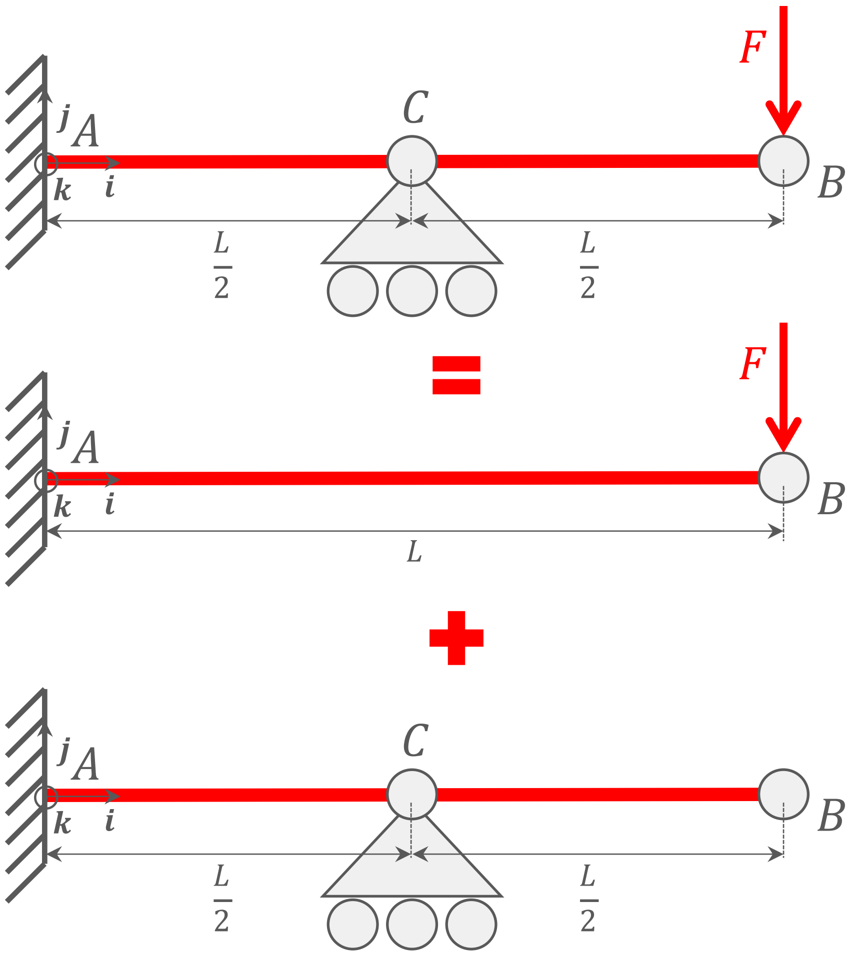
$S_T = S_1 + S_2$



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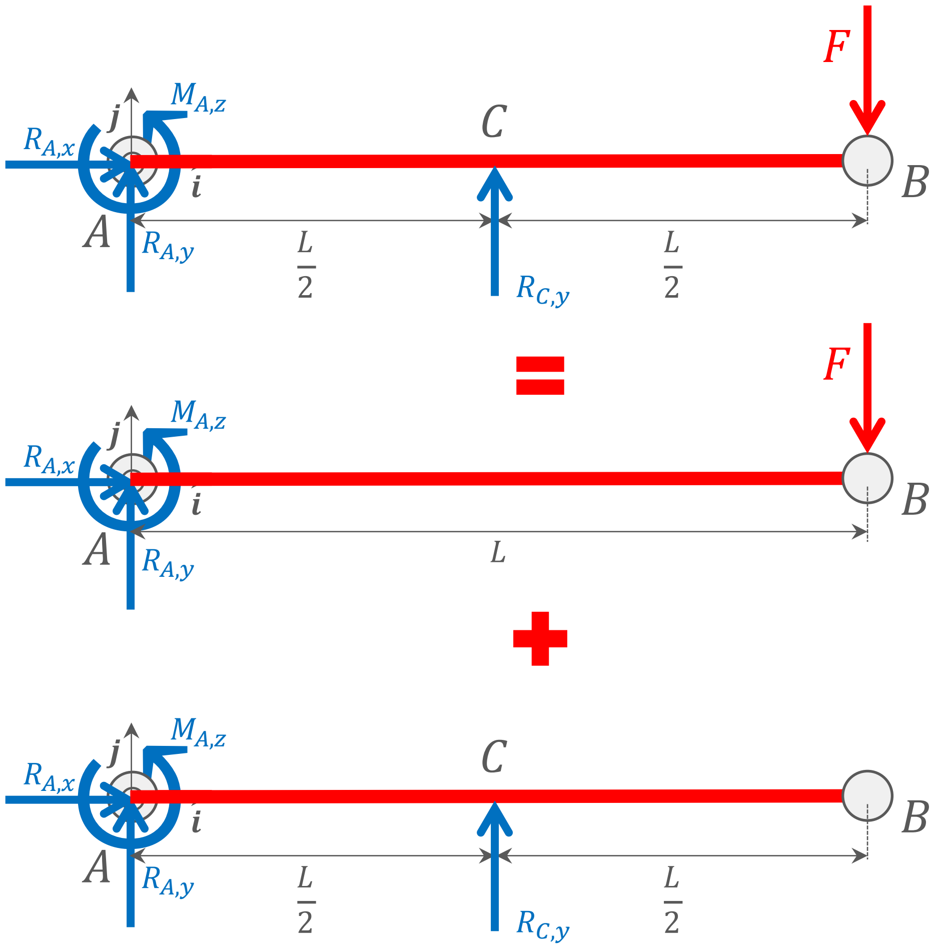
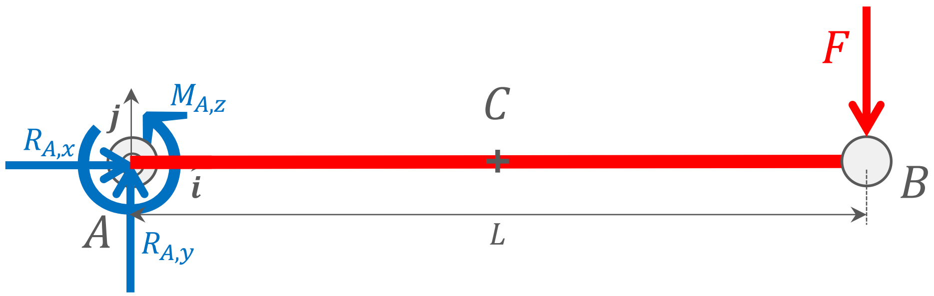


Solving S_1



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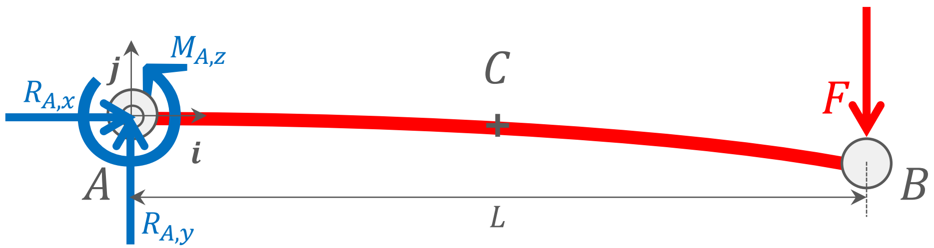

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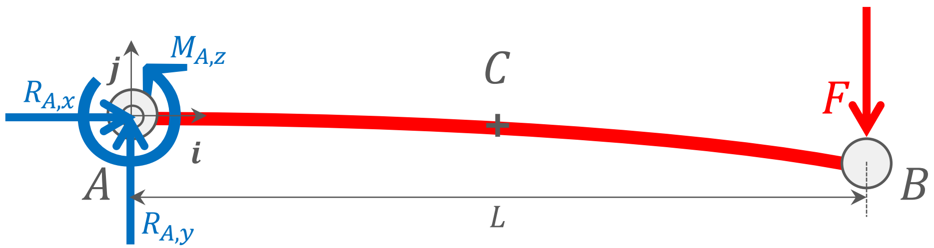

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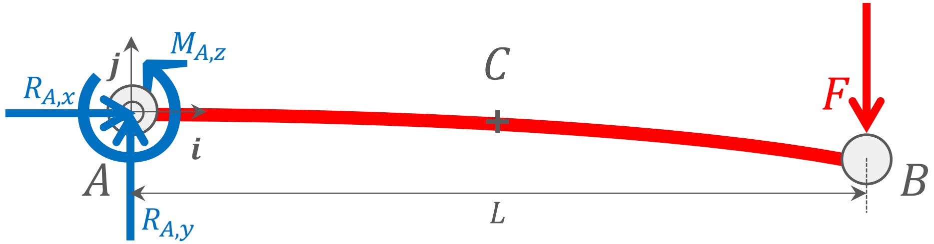

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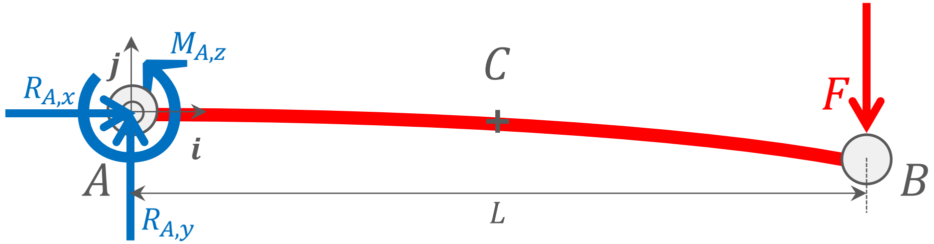

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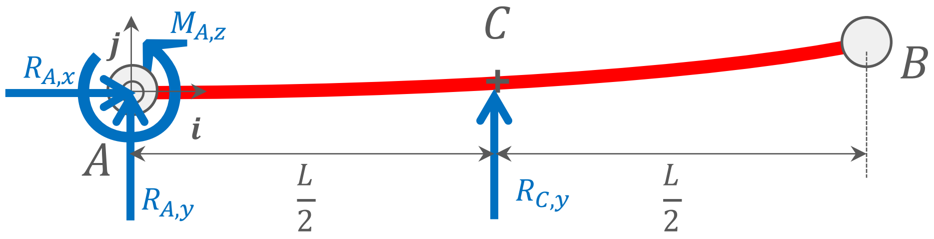

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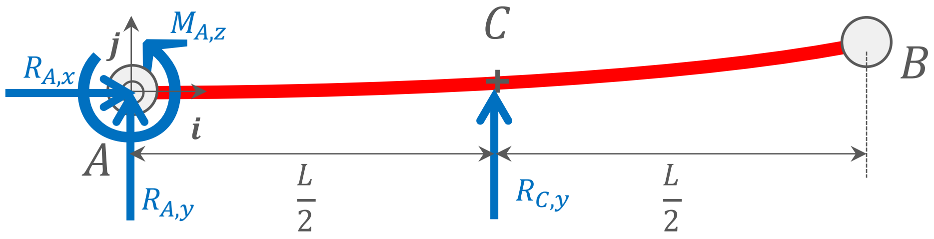

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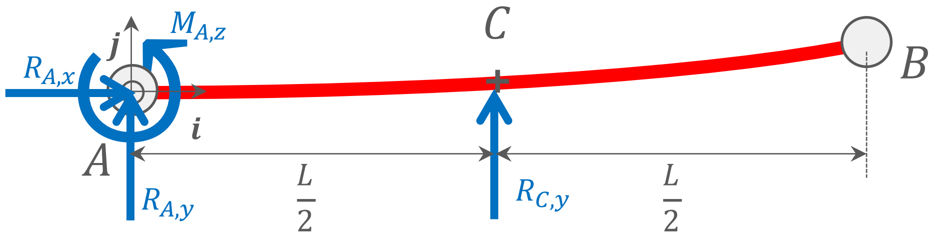

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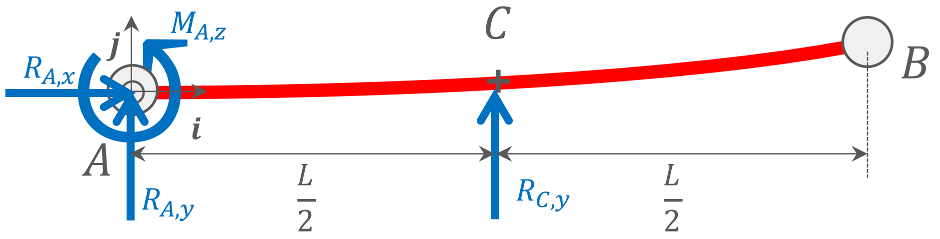

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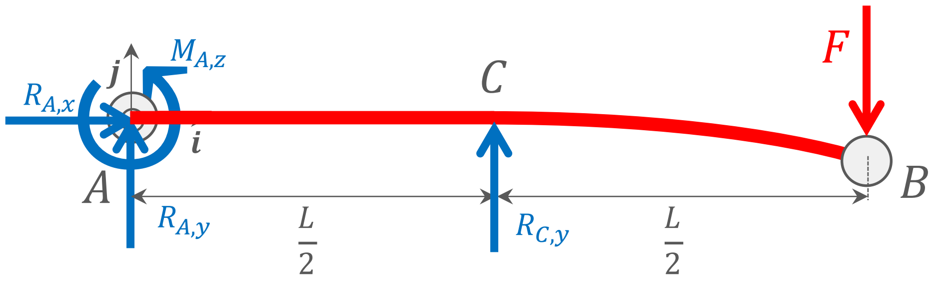

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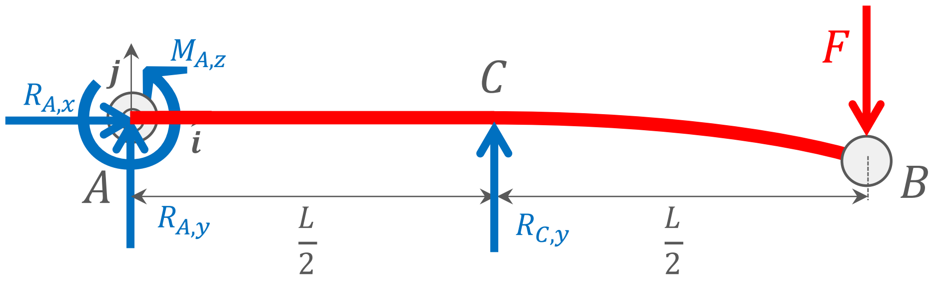

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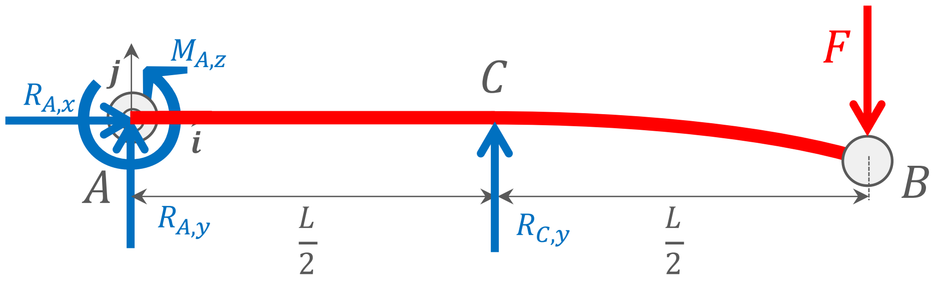

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↳ This is now a system of 3 equations with 3 unknowns
⇒ **Possible to solve!**

Thanks for your listening!

If you need further information:

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Website: www.antoine-guitton.fr

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